**Geometry of Space** Multivariable Calculus X. Du

- **Concept of Dimensions and Space**
  - One-space ($\mathbb{R}$): one dimension, one axis, one degree of freedom
    - Example: number line
  - Two-space ($\mathbb{R}^2$): two dimensions, two axes, two degrees of freedom
    - Example: Cartesian plane ($x$, $y$). Polar plane ($r$, $\theta$). Longitude & Latitude
    - Quadrants: $2^2 = 4$
  - Three-space ($\mathbb{R}^3$): three dimensions, three axes, three degrees of freedom
    - Example: Cartesian ($x$, $y$, $z$). Cylindrical ($r$, $\theta$, $z$). Spherical ($\rho$, $\theta$, $\varphi$).
    - Octants: $2^3 = 8$
  - Four-space ($\mathbb{R}^4$): four dimensions, four axes, four degrees of freedom
    - Example: Space-time ($x$, $y$, $z$, $t$) etc.
- **Parametric Equations in Space**
  - In $\mathbb{R}^3$, a curve can be expressed as a three functions of a parameter $t$, i.e. $x(t)$, $y(t)$, and $z(t)$.
  - Number of parameters is the number of free variables.
    - Points have no free variables (0 degrees of freedom). $\vec{r} = \langle x,\ y,\ z \rangle$
    - Curves may only have one free variable (i.e. one parameter)
      - Equation in three-space: $\vec{r}(t) = \langle x(t),\ y(t),\ z(t) \rangle$
    - Surfaces have two free variables (i.e. two parameters)
      - Equation in three-space: $\vec{r}(s,t) = \langle x(s,t),\ y(s,t),\ z(s,t) \rangle$
    - Solids have three free variables (i.e. three parameters)
      - Equation in three-space: $\vec{r}(r,s,t) = \langle x(r,s,t),\ y(r,s,t),\ z(r,s,t) \rangle$
  - This allows complex figures to be more easily written as a parametric equation.
  - There are an infinite number of ways to parameterize a function!
  - Trigonometric identities may help parameterize circles and ellipses.

Further notes:

- **Manifolds**
  - A topological shape that is locally Euclidean around each point.
  - Common manifolds:
    - Curve: 1-manifold
    - Surface: 2-manifold
    - Euclidean 3-space: 3-manifold
    - Minowski Spacetime: 4-manifold
  - Non-examples: figure 8, lemniscate
- ***n*-Spheres**
  - 0-sphere: two points 1-sphere: circle 2-sphere: sphere
  - Are the boundaries of $(n+1)$-disks
- ***n*-Cubes**
  - 0-cube: point 1-cube: line segment 2-cube: square
    3-cube: cube 4-cube: tesseract